Smart River Engineering

Chapter 4. Hydraulics of River Flows

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이 세상 사람들 모두 잠 들고

어둠 속에 갇혀서 꿈조차 잠이 들 때

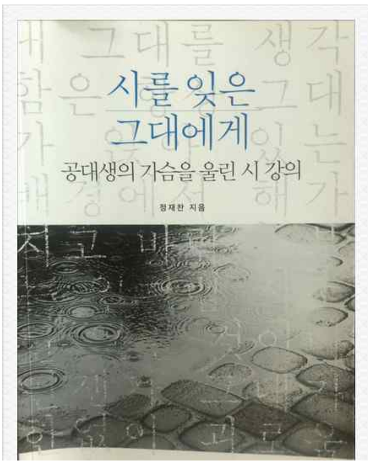
홀로 일어난 새벽을 두려워 말고

별을 보고 걸어가는 사람이 되라

희망을 만드는 사람이 되라

정호승 [희망을 만드는 사람이 되라]

중에서



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1883 Reynolds' Experiment

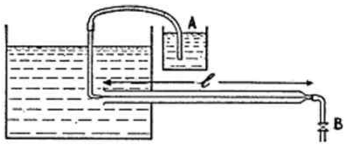


Fig. 61. Osborn Reynolds' experiment

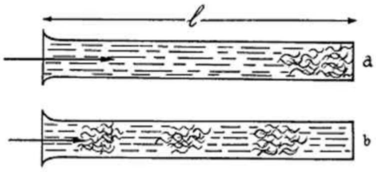


Fig. 62. Two modes of flow: laminar and turbulent

$$\mathrm{Re} = \frac{VL}{\nu}$$

Flow properties: V= characteristic velocity
L = characteristic length

Fluids property: ν = kinematic viscosity

Laminar Flows and Turbulent Flows

- Reynolds' No.

$$\mathrm{Re} = \frac{VR_h}{\nu}$$

- Ratio of inertia force to viscous force
- Characteristic velocity and length
- Critical Reynolds No. of 500 for Open-Channel Flows

Subcritical Flows and Supercritical Flows

- Froude No.

$$Fr = \frac{V}{\sqrt{gD}}$$

- (Square root of) Ratio of inertia force to gravity force
- Characteristic velocity and length

A Longitudinal Profile of an Alluvial River

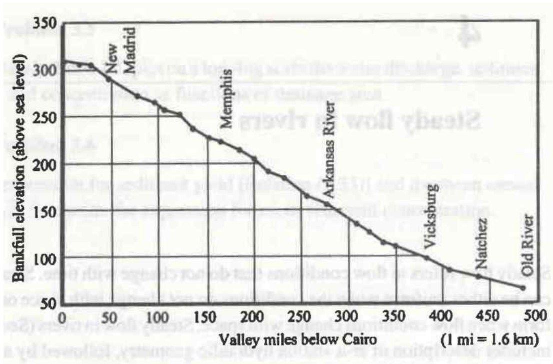


Figure 4.1. Longitudinal profile of the Mississippi River.

- Average slope = (310-70)*0.3/(485*1,600)=1/10,000
- Slope is defined to be positive.
- Left and right banks are referenced to a downstream-looking direction.

A Longitudinal Profile of a Semi-alluvial River

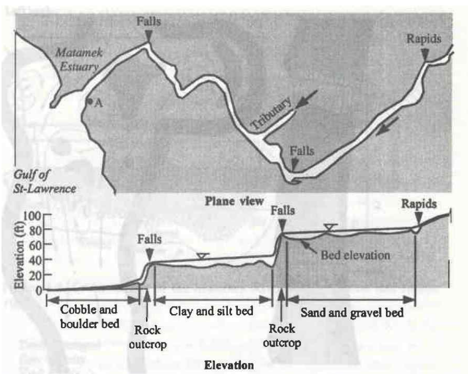


Figure 4.2. Longitudinal profile of the Matamek River (after Frenette and Julien, 1980).

- Discontinuities are seen in longitudinal profiles, bed material sizes, and flow conditions.

Geometric Properties

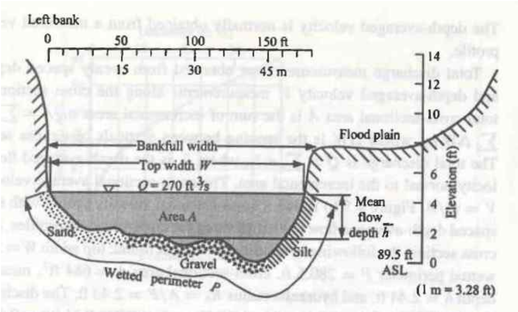


Figure 4.4. Cross section of the Matamek River (after Frenette and Julien, 1980).

- T (top width) = 53.0 m
- P (wetted perimeter) = 54.0 m
- A (cross section area) = 45.9 m^2
- D (mean flow depth) = 0.87 m
- R_h (hydraulic radius) = 0.85 m

- The mean flow depth is different from the stage.
- The hydraulic radius is very close to the mean flow depth.

Velocity Distributions

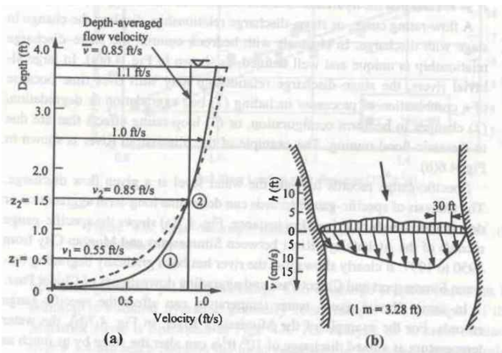
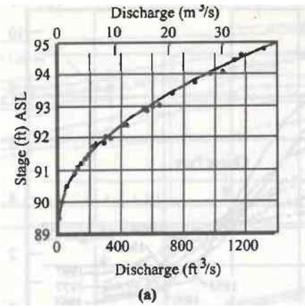


Figure 4.5. Matamek River: (a) vertical velocity profile and (b) transversal velocity profile (after Frenette and Julien, 1980).

- Q = 0.55*1+0.85*1+1.0*1+1.1*0.7=3.17 ft3/s (for T=1)
- Depth-averaged velocity = Q/3.7 = 0.86 ft/s

Rating Curve 1



- **The rating curve is the stage-discharge relationship.**
- **The rating curve is unique for channels with bedrock control.**
- **<u>The purpose of constructing the rating curve is to estimate the discharge for a certain stage</u>.**

2. Section Properties

Rating Curve 2

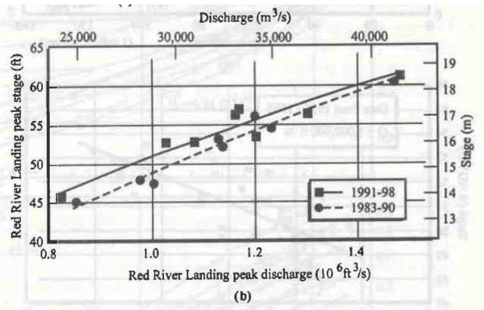


Figure 4.6. Stage–discharge relationship: (a) Matamek River and (b) Atchafalaya River.

- **The rating curve shifts over time because**
  - **- bed aggradation or degradation**
  - **- changes in bedform**
  - **- loop rating effects due to dynamic flood routing**

Hydraulic Geometry

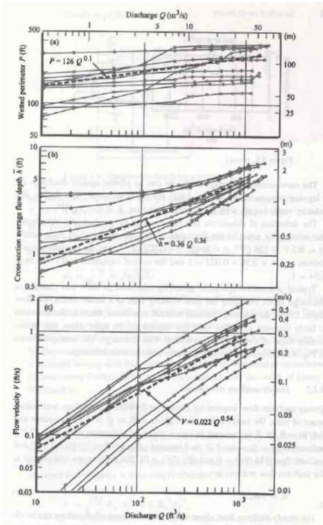


Figure 4.8. At-a-station hydraulic geometry of the Matamek River (after Frenette and Julien, 1980).

$$P = 126Q^{0.1}$$

$$h = 0.36Q^{0.36}$$

$$V = 0.022Q^{0.54}$$

- Q=WhV
- Sum of exponents =

  0.1 + 0.36 + 0.54 = 1.0

Classification of Open-Chanel Flows

- uniform flow (등류)
- non-uniform flow (부등류)

  gradually varied flow (점변류)

  rapidly varied flow (급변류)
- unsteady flow (부정류)

Uniform Flows

- The discharge, water area, velocity, and depth are constant with distance.

  velocity: velocity averaged over cross section and time

- The bottom slope (S_0), the slope of the water surface (S_w), and the slope of the energy line (S_e) are the same.

Continuity Equation

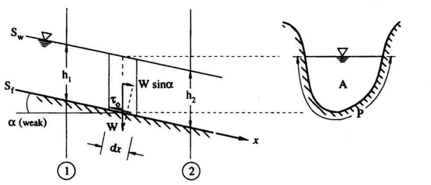


Fig. 3.2 Scheme of uniform flow.

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

- **$Q = Q_1 = Q_2$ = constant**
- **$A_1 = A_2$ = constant**
- **$H_1 = H_2$ = constant**

Momentum Equation

- A balance between the gravity & the bottom shear (friction)

$F_g = \gamma\ A\ dx\ \sin\theta$
$F_f = \tau_0\ P\ dx$

$$\boxed{\tau_0 = \gamma\ R_h\ S_0}$$

<u>This is valid regardless of the shape of the cross section.</u>

Uniform Flow Formulas

- Chezy formula

$$V = C\sqrt{R_h S_0}$$

- Manning Formula

$$V = \frac{1}{n} R_h^{2/3} S_0^{1/2}$$

- Darcy-Weisbach formula

$$V = \sqrt{\frac{8g}{f}}\sqrt{R_h S_0}$$

Dimensional Non-homogeneity

- Manning Formula

$$V = \frac{1}{n} R_h^{2/3} S_0^{1/2}$$

- What is the dimensional of n?
- Why does this happen?

Comparisons of Uniform Flow Formulas

$$\sqrt{\frac{8}{f}} = \frac{C}{\sqrt{g}} = \frac{C_m}{\sqrt{g}} \frac{R_h^{1/6}}{n}$$

- Chezy C has a dimension of $\sqrt{g}$
- It is reasonable to think that $C_m \sim \sqrt{g}$ and $n \sim L^{1/6}$
- However, in practice, we take n dimensionless and

$$C_m \sim \frac{\sqrt{g}}{L^{1/6}}$$

Ex.4.2

Velocity Distribution

- Log law for smooth-bed flows

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{u_* z}{\nu}\right) + 5.5$$

- Log law for rough-bed flows

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{z}{k_s}\right) + 8.5$$

- Keulegan's resistance relation for rough-bed flows

$$\frac{U}{u_*} = \frac{1}{\kappa}\ln\left(11\frac{H}{k_s}\right)$$

- Manning-Strickler form

$$\frac{U}{u_*} = 8.1\left(\frac{H}{k_s}\right)^{1/6}$$

Ex.4.1

Specific Energy

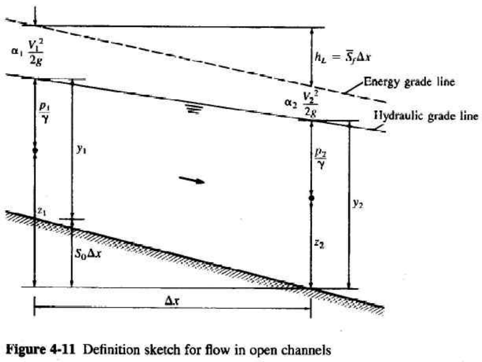


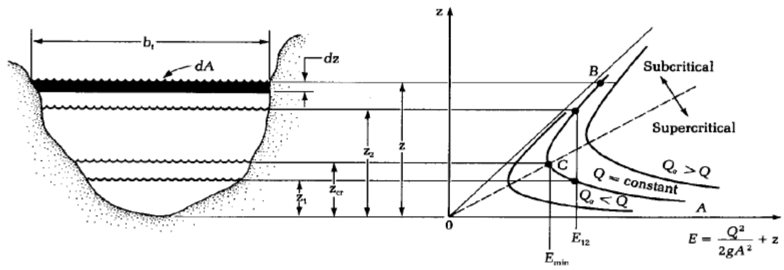
Figure 4-11 Definition sketch for flow in open channels

$$E_s = \frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2$$

- The fact is that we cannot use the energy equation because the pressure changes vertically.
- The sum of velocity head and flow depth is constant.
- The sum is defined by specific energy that is total energy per unit weight.

Specific Energy Curve

Specific-energy diagram and corresponding depths for various conditions.



- In general, for a fixed Es, two flow depths exist, alternate depths.
- A critical flow can be defined by a flow in which specific energy is minimum.

Critical Depth

- The specific energy has a minimum value under which the given Q cannot occur.

$$\frac{dE_s}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy} = 1 - \frac{V^2}{gA}\frac{dA}{dy}$$

Note that dA/dy ≈ T. Then, we have

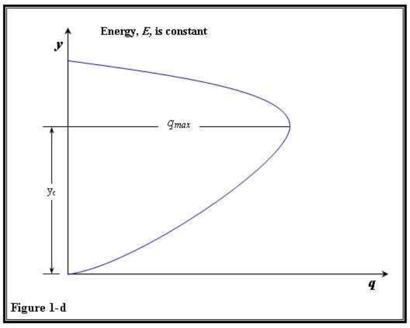
$$\frac{dE_s}{dy} = 1 - \frac{V^2 T}{gA} = 1 - \frac{V^2}{gD}$$

where D = hydraulic depth.

- Critical flow occurs when the specific energy is minimum.

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad Min(E_s) = \frac{3}{2} y_c$$

Flow Depth vs. Discharge Curve



$$Q = \sqrt{\frac{2g}{\alpha}(E_s - y)B^2 y^2}$$

$$y_c = \frac{2}{3}E_s$$

- For a fixed Es, critical flow is made when the discharge is maximum.

Froude Number (1)

- The Froude number is defined by

$$Fr = \frac{V}{\sqrt{gD}}$$

- Square root of the ratio of the inertia force to the gravity force (Can you show this?)
- The most important dimensionless number for the open-channel flow.
- Note that the characteristic length is the hydraulic depth. (How about for the Reynolds number?)

Froude Number (2)

- Classification of open-channel flows
  - subcritical flow: Fr < 1
  - critical flow: Fr =1
  - supercritical flow: Fr > 1
- Critical Slope: The slope of the channel that sustains a given discharge at a uniform and critical depth.

$$S_c = \frac{n^2 gD}{R_h^{4/3}}$$

- Note that the uniform flow on a mild slope is subcritical, but the uniform flow on a steep slope is supercritical.

Ex.4.3

Specific Force: 比力

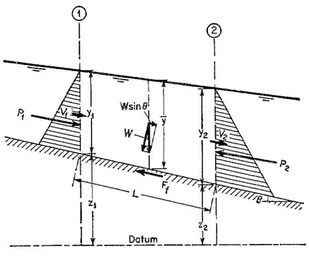


Fig. 3-7. Application of the momentum principle.

$$\sum F = \rho Q\left(V_2 - V_1\right)$$

$$\sum F = P_1 - P_2 + W\sin\theta - F_f$$

if the gravity and friction are ignored,

$$\frac{Q^2}{gA_1} + h_{G1}A_1 = \frac{Q^2}{gA_2} + h_{G2}A_2$$

$$F_s = \frac{Q^2}{gA} + h_G A$$

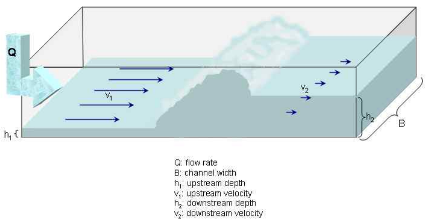
- The force per unit weight of water is conserved.

Hydraulic Jump

- What is the role of the hydraulic jump?

5. Hydraulic Jump

Hydraulic Jump



$$V_1 y_1 = V_2 y_2$$

$$\frac{\gamma y_1^2}{2} - \frac{\gamma y_2^2}{2} = \rho q \left(V_2 - V_1\right)$$

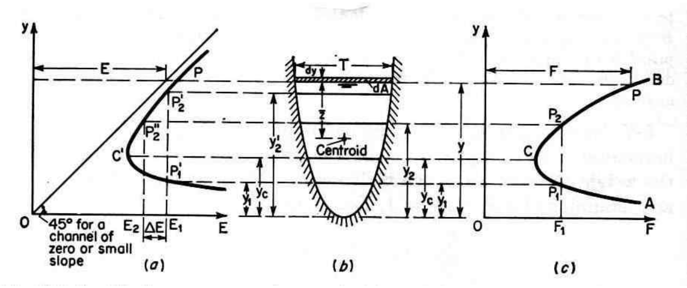
$$\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + h_L$$

$$y_2 = -\frac{y_1}{2} + \frac{y_1}{2}\sqrt{1 + 8Fr_1^2}$$

$$h_L = \frac{\left(y_2 - y_1\right)^3}{4 y_1 y_2}$$

- Three equations and three unknowns

Specific Force Curve



- Two depths of the same Es exist (alternate depths).
- However, the sub-critical flow after the hydraulic jump has Es – ΔE.
- Two depths of the same specific force are conjugate depths.

Ex.4.4

Gradually Varied Flows

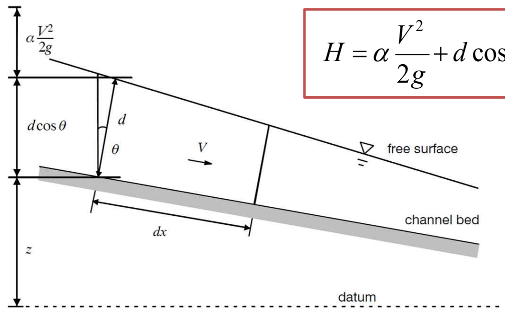
- Features
  - steady flows the water surface of which changes gradually
  - so the pressure change hydrostatically (parallel streamlines)
- Governing equation:

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - Fr^2}$$

here S_0 = bed slope, S_e = slope of energy line, and

$$Fr^2 = \frac{V^2}{gD} = \frac{Q^2 T}{gA^3} = fn(y)$$

Backwater Equation 1



$$H = \alpha \frac{V^2}{2g} + d\cos\theta + z$$

$$\frac{dH}{dx} = -S_e$$

$$\frac{dz}{dx} = -S_0$$

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = -\frac{Q^2}{gA^3}\frac{dA}{dy}\frac{dy}{dx} = -\frac{Q^2 T}{gA^3}\frac{dy}{dx} = -\frac{V^2}{gD}\frac{dy}{dx} = -Fr^2\frac{dy}{dx}$$

6. GVF

Backwater Equation 2

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - Fr^2}$$

- Non-linear ODE
- dy/dx = 0 when $S_0 = S_e$, indicating the uniform flow.
- Valid for arbitrarily-shaped channel section

- The bank elevation is determined from the stage for the designed flow by calculating the backwater equation.

시간에 따라 급격히 유량 혹은 수위가 변하는 경우를 제외하고는 (실시간 홍수예경보), 대부분의 홍수 문제는 배수곡선식으로 해결된다.

Backwater Equation 3

$$\frac{dy}{dx} = S_0 \frac{1-\left(y_n / y\right)^{10/3}}{1-\left(y_c / y\right)^3}$$

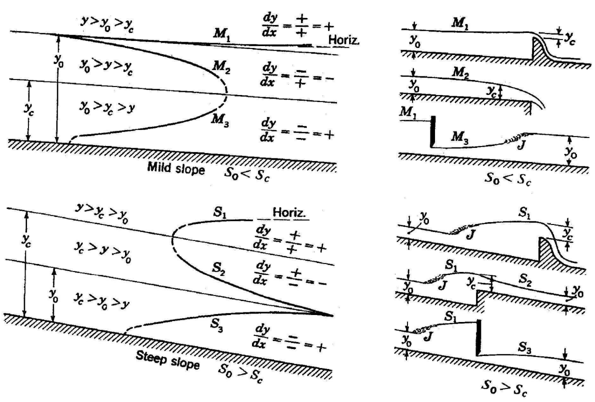
where

$$S_0 = \frac{n^2 Q^2}{b^2 y_n^{10/3}} \qquad S_e = \frac{n^2 Q^2}{b^2 y^{10/3}} \qquad Fr^2 = \left(\frac{y_c}{y}\right)^3$$

for a wide rectangular channel.

• Note that Manning equation (uniform flow formula) is used to estimate the slope of the energy line. How?

Gradually Varied Flows: M & S Curves



6. GVF

Gradually Varied Flows: C, H, & A Curves

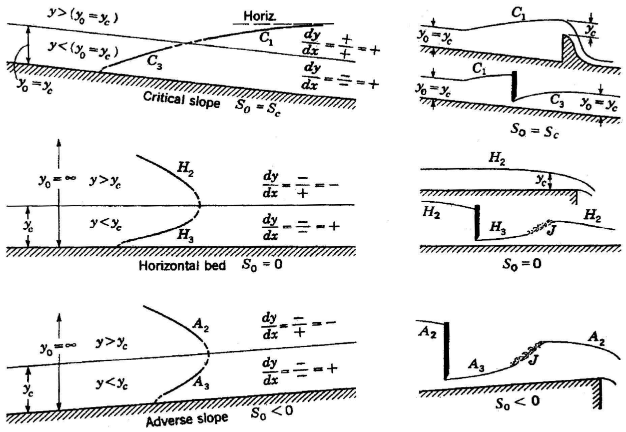
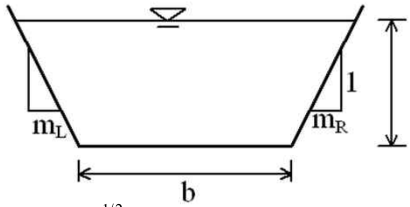


Figure 11.20 Various types of nonuniform flow with flow from left to right.

Homework

Normal Depth & Critical Depth (1)

- **Derive the formulas to calculate the critical depth and normal depth of the trapezoidal channel with different side slopes.**



$$\frac{Q_c}{g^{1/2}} = \left\{ \frac{\left[bh_c + \frac{(m_L+m_R)}{2} h_c^2 \right]^3}{b+(m_L+m_R)h_c} \right\}^{1/2}$$

$$\frac{nQ_n}{S^{1/2}} = \left\{ \frac{\left[bh_n + \frac{(m_L+m_R)}{2} h_n^2 \right]^5}{\left[b+\left(\sqrt{1+m_L^2}+\sqrt{1+m_R^2}\right)h_n \right]^2} \right\}^{1/3}$$

Homework

Normal Depth & Critical Depth (2)

- Obtain the critical depth and normal depth of the channel whose
  - n = 0.02
  - S = 0.0001
  - Q = 20,000 cms
  - m_L = 2.5
  - m_R = 3.0
  - b = 1,000 m

You may use Excel(목표값 찾기 기능) or MatLab (not your calculator).